Dynamically applied loads are balanced by elastic and inertial forces. Given small plate dimensions ($\xi_* < 2$), elastic forces predominate. At $\xi_* \to \infty$, the plate resists only with inertial forces, and $\varphi(0) \to 1$. For short plates ($\xi_* < 3$), the maximum values of M_1 and M_2 are seen at the center. Here, M_2 changes slightly (up to 10%) along the radial section to the value r = 0.6R. Only at the support does it turn out to be 40% less than at the center of the plate. With an increase in the radius of the plate, strains begin to be localized next to the supports. Thus, the maxima of M_1 and M_2 are displaced from the plate's center toward the support. Figure 7 shows the distribution along the plate radius of quantities characterizing bending elements. Here $\xi_* = 7.05$.

LITERATURE CITED

1. A. A. Il'yushin, Plasticity [in Russian], Gostekhteorizdat, Moscow—Leningrd (1948).
2. V. V. Novozhilov, Theory of Elasticity [in Russian], Sudpromgiz, Leningrad (1958).
3. G. Cauderer, Nonlinear Mechanics [Russian translation], IL, Moscow (1961).
4. L. E. Sedov, Similarity and Dimensional Methods in Mechanics, Nauka, Moscow (1977).
5. L. V. Ovsyannikov, Group Analysis of Differential Equations [in Russian], Nauka, Moscow (1978).
6. L. G. Donnel, Beams, Plates, and Shells [in Russian], Nauka, Moscow (1982).
7. G. Korn and T. Korn, Mathematical Handbook for Scientists and Engineers, 2nd ed., McGraw-Hill, New York (1967).
8. A. I. Lur'e, Operational Calculus and Its Application to Problems of Mechanics [in Rusaian], Gostekhizdat, Moscow—Leningrad (1951).
9. S. A Zegzhda and M. I. Ulitin, "Comparison of vibrations of finite and infinite plates in transverse impact," Prikl. Mekh., No. 2 (1975).
10. L. N. Aleshin, I. A. Prikhod'ko, and V. I. Sebekina, "Stress-strain state of an infinite elastic plate subjected to shock loading, in: Studies in the Theory and Methods of Structural Design [in Russian], TsNII Stroit. Konstruktsii, Moscow (1984).
11. S. P. Timoshenko and S. Woinowsky-Krieger, Theory of Plates and Shells, McGraw-Hill, New York (1959).
12. V. P. Yastrebov, "Degenerate similarity solutions in dynamic problems of strain calculation in nonlinearly elastic bodies," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1988).

MECHANICAL MODEL OF AN ELASTOPLASTIC BODY

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UDC 539.3

It is presently held that the phenomenon of the plastic deformation of solids is based on the shear or slip of one part of the material over another [1-12]. Despite the agreement on the nature of plastic deformation, different approaches have been taken to describe the phenomenon. One school of thought is that plastic deformation is governed by a shearing process which takes place in a whole fan of slip planes [1-3]. Other investigators [4-12] believe that such a process occurs only in a finite system of slip planes with a particular orientation. In [4-8], this system was associated with the set of planes acted upon by the principle shear stresses. Another set was hypothesized to be composed of equally-inclined or octahedral planes [9, 10]. As regards macroscopic studies, they do not contradict any of the approaches taken [3, 5, 13, 14], but they do show that the last-mentioned methods have certain advantages: the beginning of plastic deformation is described best by the condition of constancy of the octahedral shear stress (or von Mises condition) [12, 15]; during simple loading, the "single" curve hypothesis, establishing the dependence of the octahedral shear strain on the octahedral shear stress [16-18], turns out to be valid. Microscopic studies undertaken to determine the planes of slip in a body being deformed also have failed to resolve the problem of selecting an approach. This is because actual materials are to a known

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degree locally inhomogeneous and anisotropic in terms of ductility properties [19, 20], while the above-mentioned approaches are phenomenological.

The studies cited already analyze the purely plastic state of a deformed body — the total strains are divided into elastic and plastic components. However, there have been investigations [21, 22] which have focused on how a solid is elastically deformed. In particular, researchers have posed the question: what is the Poisson effect in the case, for example, of the tension or compression of a straight rod when no forces act in the transverse direction but strain is still present? Different schemes of deformation have been suggested to explain this phenomenon. However, no link has been established between the mechanisms of elastic and plastic deformation. Yet it is clear that they should be interdependent.

The present study has the goal not on setting these two deformation processes in opposition to one another, but of treating them as component parts of a single process — elastoplastic deformation. Such a formulation of the problem makes it possible to take the necessary approach toward the study of plastic strains and to establish a mechanism of elastic deformation of solids which is continuous and which naturally changes into the mechanism of elastoplastic deformation.

We construct a mechanical model of a solid undergoing deformation. The model's structure is similar to that of Rubik's cube — its component elements are rigid nondeformable blocks connected by elastic springs. Tension or compression of the springs causes an increase or decrease in the volume of the model of the medium, while slip of the blocks relative to one another results in a change in the shape of the model. In the case of small strains, the model is deformed in accordance with the normal Hooke's law. In the case of more substantial strains, it satisfies the generally-accepted laws of plastic deformation: its volume changes elastically, while the plastic shears are independent of the hydrostatic pressure. In essence, for any external load the model undergoes only two types of deformation — simple shear and extension. This makes the model self-consistent, since simple shear and extension take place without manifestation of the Poisson effect.

To construct a mechanical model of a medium, it is necessary to specify its structure and mechanism of deformation. Chernov—Lüders lines have been seen in many tests on the surfaces of tested materials. Their appearance is linked with the attainment of appreciable plastic strains in the specimen [23, 24]. Chernov-Lüders lines are traces of surfaces of strain localization that are formed in the body. Intersecting one another, these surfaces impart a blocklike structure to the medium. The blocks in specimens slide over one another and break up due to the formation of new planes of weakness in the material [25]. The empirically-observed phenomenon of strain localization and its characteristics best suggest the structure of the model of the medium and the mechanism of its deformation. The model should consist of blocks whose main mechanism of deformation is slip of the blocks over one another. This conclusion was reached in [4-6] in studies of the plastic deformation of initially uniform and isotropic materials.

Several methods have been proposed for determining the block structure of strain-hardening materials: one of them identifies this structure with the grid of characteristic lines of an ideally plastic body [26], while others have devised special criteria for strain localization [23, 27, 28] which in some manner conform to the condition of hyperbolicity of the main system of differential equations. According to the hypotheses in [4-6], the blocks in a uniformly deformed strain-hardening material are cut by planes on which the shear stresses are maximal and exceed the elastic limit of the material. In all of the above-cited studies, the blocks are deformed elastically and the plastic-strain components are formed by the slip of some blocks over others.

Let us critically examine the studies [4-8, 26]. First of all, it should be noted that the block structure of the model of the medium is merely a hypothesis. It has not been found to be valid on the basis of study of the governing relations of the medium. It is assumed that for its construction it is sufficient to limit oneself to analysis of the stress state, without involving these governing relations. Generally speaking, there may be no time correspondence between the relations and the slip planes (even if these are the planes of action of the maximum shear stresses). In this sense, it is preferable to consider the conclusions reached in [23, 27, 28], where investigators proposed strain localization criteria that take into account the elastoplastic properties of the medium. However, these criteria also ultimately yield little information.

Calculations show that the type of system of basic equations that describes strain-hardening materials is elliptic and, in accordance with the above criteria, no strain localization takes place. Calculations have been performed using the classical theories – the strain theory of plasticity and the theories of plastic flow with the Tresca and von Mises yield conditions. The results were the same in every case. This fact has stimulated many researchers to reexamine the basic principles underlying the construction of the governing relations of the theory of elastoplastic strains and to change them. In particular, it has been suggested that the principle that plastic strains are of a gradient character be abandoned [27, 29]. The resulting systems of differential equations become hyperbolic for certain values of the input parameters [28, 30], but the governing relations in these systems have an intrinsic contradiction – one of them consists of the absence of a continuous transition from the region of additional plastic loading to the region of elastic unloading. At the same time, the fact of strain localization or the existence of a block structure are overlooked even by the classical theories and are expressed not in the basic system of differential equations, but through the governing relations of the medium – which reflect the behavior of its mechanical model with uniform deformation. It was concluded later in [4-8] that the plastic shears on slip planes (more accurately, their increments) depend not only on the natural shear stresses but also on the shear stresses in other systems of planes. This situation was explained in [4, 5] as being the result of anisotropy of the plastic state. In fact, it only legitimizes the paradox due to the Poisson effect. A third observation pertains to the fact that plastic deformation of the medium is considered apart from elastic deformation.

Proceeding on the basis of the same experiments involving strain localization (these tests having shown how materials are deformed in general, not just plastically), we propose that the elastic deformation of a medium is also based on shear or elastic (reversible) slip of one block over another. If we assume that elastic slip of the material occurs on planes with the normal $\mathbf{n}_1$ and that plastic slip occurs on planes with the normal $\mathbf{n}_2$, , where $\mathbf{n}_1 \neq \mathbf{n}_2$, then one readily comes up against a contradiction: no matter how great the load, elastic shear can never change into plastic shear, plastic shear will never be preceded by elastic shear (no matter how small the elastic strain). To avoid such contradictions, it is necessary that the process of plastic deformation of the medium be compatible with the elastic mechanism. Meanwhile, the slip planes or the block structure of the medium must be determined by studying its elastic state [31, 32, 12].

Considering the above remarks, we hypothesize that the block structure in an initially homogeneous and isotropic material be cut up not in the plastic state – as proposed in [4-8, 23, 28] – but even before the formation of elastic strains in the material.* The situation is roughly as follows. First a block structure is formed in the material under the influence of the applied forces. Then the blocks begin to move relative to one another, and the material is deformed as a result of their motion. In the mechanical model of the medium, the blocks are rigid and nondeformable. Reversible movement of the blocks corresponds to elastic deformation of the medium, while their irreversible movement corresponds to inelastic deformation. The block structure is influenced by the stress state – with rotation of the principal axes of the stress tensor, the old block system is closed up and filled with material, forming a new system. The block structure also depends on the structure of the medium itself. Thus, we will assume that is a priori unknown. Keeping in mind that the choice of the latter will influence the governing relations of the medium in both the elastic and plastic regions,† we will attempt to solve the inverse problem – establish the block structure from known governing relations of the medium, such as Hooke' law.

To solve the above-stated problem, it is necessary to consider the question of the mechanism responsible for deformation of the block structure. We present the following situation. Let there be a certain set of rigid nondeformable blocks which are somehow connected to one another (Fig. 1a). Having subjected this system to a rigid rotation, we apply normal σ and shearing τ forces to it. Under the influence of the forces σ and τ the given system may generally be deformed as shown in Fig. 1b-d. Let us take a more detailed look at the schemes of deformation that have been discussed here. In the first case (Fig. 1b), the

*This hypothesis conforms to a mechanical model of the medium which is a phenomenological reflection of the actual material.
†The block structure can thus be regarded as a characteristic of the material.

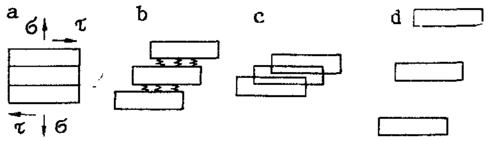

Fig. 1

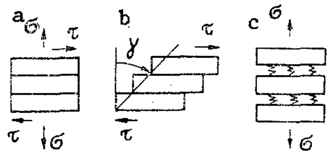

Fig. 2

shearing force τ causes the blocks to slide over one another by a certain relative amount γ (Fig. 2b). Under the influence of the force σ they are separated in the orthogonal direction by a relative amount ε so that an elastic connection is formed between them. We specify the latter through the relation $\varepsilon = \sigma/K$, where ε characterizes the strain corresponding to simple extension [33] in the direction n. The value of K (a material constant) is determined by the stiffness of the elastic springs connecting the blocks (Fig. 2c). In the second case (Fig. 1c), slippage of the blocks is followed by the penetration of one block by another. In the third case (Fig. 1d), the blocks are separated so that nothing connects them – the material is divided into separate parts.

What distinguishes the scheme of deformation in Fig. 1b from the schemes in Fig. 1c and Fig. 1d? First of all, here each force τ and σ causes only its natural strain, i.e., there is no Poisson effect. Also, the block structure remains compatible connected after deformation, which is guaranteed by the condition of elastic change in the strain ε; this structure can be further deformed. By the mechanism of deformation of the block structure, we mean this scheme of deformation (Fig. 2). The relation $\sigma = \sigma(\varepsilon)$ is linear, and curve of $\tau = \tau(\gamma)$ for elastoplastic deformation is shown in Fig. 3.

This curve reflects the dry friction between particle-blocks of the material as they slip past one another. This was noted in [34]. The fact that the curve has the same form at $\sigma > 0$ (tension) and $\sigma < 0$ (compression) for many metallic materials suggests that forces associated with internal interaction of the particles of the material (interatomic, intermolecular, interstructural forces) play a more important role in the slip planes than does σ. The attraction of the particles to one another due to these forces is much greater than the tension or compression of the particles by the forces σ from external loading. We will use N to denote the contribution of the interaction forces to the pressure on the slip planes. Then the total pressure is equal to $N + \sigma$. Let the roughness of the surfaces of sliding blocks be determined by the friction coefficient k_{fr}. We will examine the plastic deformation of the material. For plastic deformation to begin, it is necessary that the shear stress on the slip planes exceed the friction force, i.e., we must have* $\tau \geq k_{fr}(N + \sigma)$. It follows from this that for $|N| \gg |\sigma|$ and $|k_{fr}\sigma| \ll \tau$, the curve $\tau = \tau(\gamma)$ will actually be independent of σ. Meanwhile, $k_{fr}N$ can be identified with the elastic limit of the material $\tau_s = k_{fr}N$. It is not hard to see that due to the surface roughness of the sliding blocks, the material also undergoes elastic deformation. Before the blocks undergo plastic slip, the different types of projections along their contact surfaces are stressed, and their deformation determines the elastic deformation of the medium in shear.

The following question arises: if the diagram $\tau = \tau(\gamma)$ reflects dry friction between particles of the material as they slip, then why is this accounted for twice in theoretical constructions? This was done, for example, in [8, 11, 35].†

Now let us turn to finding the block structure or slip planes in an initially uniform and isotropic medium. We subject a specimen of this material to uniform loading. Let the loading be such that the material is deformed elastically. We determine the principal axes of the stress tensor and designate them as 1, 2, 3. In the coordinate system connected with these axes, we construct the stress vector $\mathbf{p}$ and strain vector $\mathbf{q}$ on an arbitrarily oriented plane with a unit normal $\mathbf{n}$. According to the Cauchy formula, $\mathbf{p} = \sigma_1 n_1 \mathbf{e}_1 + \sigma_2 n_2 \mathbf{e}_2 + \sigma_3 n_3 \mathbf{e}_3$ (n_1, n_2, n_3 are direction cosines of $\mathbf{n}$, $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are unit vectors of the coordinate system 1, 2, 3), while $\mathbf{q} = \varepsilon_1 n_1 \mathbf{e}_1 + \varepsilon_2 n_2 \mathbf{e}_2 + \varepsilon_3 n_3 \mathbf{e}_3$. The strain vector $\mathbf{q}$ characterizes the relative displacement of the plane with the normal $\mathbf{n}$ when its rigid rotation is fixed.

*In the orthogonal direction $\mathbf{M} + \varepsilon = (\mathbf{N} + \sigma)/K$ (the strain $\mathbf{M}$ is caused by the force $\mathbf{N}$). Assuming that $\mathbf{M} = N/K$, we obtain $\varepsilon = \sigma/K$.

†To verify this, it is sufficient to set the friction angle $\varphi_* = 0$ in [8, 11, 35] and examine the elastoplastic diagram in the coordinates shear stress–shear strain.

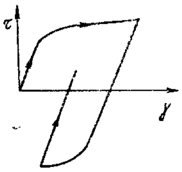

Fig. 3

We will examine the projection of these vectors on the normal $\mathbf{n}$ and in the direction tangent to the plane. For the normal components we have

$$\sigma_n = \mathbf{p}\cdot\mathbf{n} = \sigma_1 n_1^2 + \sigma_2 n_2^2 + \sigma_3 n_3^2, \quad \varepsilon_n = \mathbf{q}\cdot\mathbf{n} = \varepsilon_1 n_1^2 + \varepsilon_2 n_2^2 + \varepsilon_3 n_3^2. \quad (1)$$

The tangential projections are determined by the equalities

$$\boldsymbol{\tau}_n = \mathbf{p} - \boldsymbol{\sigma}_n, \ \boldsymbol{\gamma}_n = \mathbf{q} - \boldsymbol{\varepsilon}_n \ (\boldsymbol{\sigma}_n = \sigma_n \mathbf{n}, \ \boldsymbol{\varepsilon}_n = \varepsilon_n \mathbf{n}). \quad (2)$$

For convenience, we introduce the notation [4] $T_{13} = (\sigma_1 - \sigma_3)/2$, $\sigma_2' = \sigma_2 - (\sigma_1 + \sigma_3)/2$. Then

$$\boldsymbol{\tau}_n = T_{13}\left[n_1\mathbf{e}_1 - n_3\mathbf{e}_3 - \left(n_1^2 - n_3^2\right)\mathbf{n}\right] + \sigma_2' n_2 (\mathbf{e}_2 - n_2\mathbf{n}). \quad (3)$$

Now we use the relations of Hooke's law. Having inserted them into (1)-(2), we express the normal and tangential components $\mathbf{q}$ through the projection $\mathbf{p}$:

$$\varepsilon_n = \frac{1-2\nu}{E}\sigma_n + \frac{3\nu}{E}\left[T_{13}\left(n_1^2 - n_3^2\right) + \sigma_2'\left(n_2^2 - \frac{1}{3}\right)\right], \quad \boldsymbol{\gamma}_n = \frac{\boldsymbol{\tau}_n}{2\mu}. \quad (4)$$

We attempt to select planes in the material which first of all are independent of the method of specification of the forces $\boldsymbol{\sigma}_n$, $\boldsymbol{\tau}_n$, i.e., are independent of σ_n, T_{13}, σ_2' and their ratios. Secondly, the planes should be chosen so that the normal strain $\boldsymbol{\varepsilon}_n$ on them is caused only by the normal force $\boldsymbol{\sigma}_n$, and the shear strain $\boldsymbol{\gamma}_n$ is caused only by the shearing force $\boldsymbol{\tau}_n$. This is how we want to approximate the scheme (Fig. 2) we adopted earlier. Comparing (3) and (4), we become convinced* that the sought planes coincide with the planes equally inclined to the axes 1, 2, 3. There are eight such planes, and together they form a regular octahedron [9, 36]. Hooke's law has the following form for the octahedral planes

$$\boldsymbol{\varepsilon}_n = \boldsymbol{\sigma}_n/K, \ \boldsymbol{\gamma}_n = \boldsymbol{\tau}_n/(2\mu). \quad (5)$$

Equations (5) conceal a certain ambiguity – it can be assumed that the forces $\boldsymbol{\sigma}_n$ cause transverse strains, but it can also be hypothesized that no such strains are formed. In light of the duality of the situation and the desire to avoid the paradox caused by the Poisson effect, we assume that the normal forces $\boldsymbol{\sigma}_n$ in system (5) do not cause transverse strains. In this case, the first equation of (5) determines simple extension, while the second determines simple shear. It follows from a comparison of the scheme (Fig. 2) and the formulas (5) that the block structure of an initially uniform and isotropic medium should be formed by octahedral planes. Since Eqs. (5) are valid in each of the four families of octahedral planes, it is natural to expect that the block structure will be cut up into four systems of octahedral planes and that in each such family the material will be deformed independently and in accordance with the scheme in Fig. 2.

As an illustration, Fig. 4 shows a diagram of the tension of a rectangular beam in plane strain. In contrast to the previous case, the material here has been divided not into octahedral figures but rectangular prisms. The generating planes of the prism are parallel to the planes of action of the maximum shear stress. The same scheme is applicable to the deformation of an initially uniform and isotropic medium [24]. Plane strain is a constrained type of deformation [5], and the Hooke's law relations for plane strain become the equations

*At $n_1 = 0$, $n_2 = 0$, $n_3 = 0$, the strain ε_n depends on the normal stresses on other planes. For ε_n to be independent of τ_n at $n_1 \neq 0$, $n_2 \neq 0$, $n_3 \neq 0$, it is necessary that the direction cosines of the normal satisfy the equations $n_1^2 - n_3^2 = 0$, $n_2^2 - 1/3 = 0$, $n_1^2 + n_2^2 + n_3^2 = 1$. It follows from this that $n_1 = \pm 1/\sqrt{3}$, $n_2 = \pm 1/\sqrt{3}$, $n_3 = \pm 1/\sqrt{3}$.

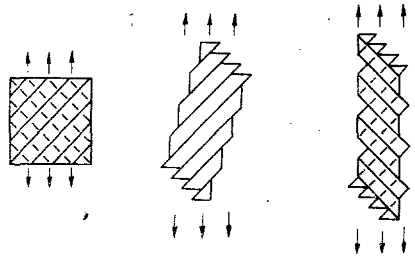

Fig. 4

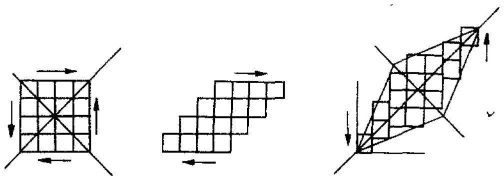

Fig. 5

$$\varepsilon_1 + \varepsilon_3 = (\sigma_1 + \sigma_3)/k', \ \varepsilon_1 - \varepsilon_3 = (\sigma_1 - \sigma_3)/(2\mu), \ \varepsilon_2 = \sigma_2/C. \tag{6}$$

If we set $C = \infty$ in these equations (if we assume that the material is rigid in the direction 2), put $k' = K/(1 + \nu)$, and always maintain $\sigma_2 = \nu(\sigma_1 + \sigma_3)$, then we will immediately have the Hooke's law relations for plane strain. The character of Eqs. (6) reflects the prismatic structure of the medium. The first equation in (6) reflects the elastic change in ε_n on the planes of action of the maximum shear stress ($\varepsilon_n = \sigma_n/k'$), the second equation reflects simple shear, and the third reflects simple extension in the direction orthogonal to the plane of the figure (it is equal to zero, since $C = \infty$). The prisms slide along one another – first in one family of planes and then, by virtue of the pairing of the shear stresses, in another family. As a result, we obtain the pattern of deformation shown in Fig. 4.

This example shows that in the case of constrained deformation, the structure of an initially uniform and isotropic material may be rearranged – rectangular prisms are formed instead of the above-mentioned octahedra. Both cases may be characterized by the same form of the curve $\tau = \tau(\gamma)$, which characterizes the resistance of the medium to elastoplastic shears. This hypothesis must be verified experimentally.

Now let us return to analysis of the Poisson effect. As can be seen, Eqs. (5) and (6) do not include the Poisson effect. Then how is it to be manifest? In the case of shear, there is not only a change in the angles of an element of the medium (Fig. 5), but a decrease in one of its diagonals and an increase in the other. The first causes extension of the material, the second causes contraction. In shear, both of these phenomena occur together (one does not exist without the other). It follows from this that the Poisson effect is of a purely geometric character. It is formed exclusively as a result of shear, although it is not manifest during the shearing process itself.

The question of the block structure of an initially uniform and isotropic medium has thus been resolved. In the general case, the mechanical model of a uniformly deformed solid consists of rigid nondeformable blocks formed by the intersection of octahedral planes. The model of the body experiences only two types of strain – simple shear and extension.

Now let us establish the character of deformation of the mechanical model on the basis of existing theories of elastoplastic deformation of materials. We will take the strain theory of plasticity. In accordance with this theory, the mechanical model of a medium will be deformed in accordance with the rule

$$\varepsilon_n = \sigma_n/K, \ \gamma_n = \tau_n/(2\mu_c), \ \mu_c = \mu_c(\tau_n). \tag{7}$$

The plastic shear γ_n^p depends on the value of τ_n reached on the octahedral planes and occurs in the direction of its action. Simple extension takes place in the directions normal to the octahedral planes. The yield condition of the model τ_n = const coincides to within the numerical multiplier with the von Mises condition.

Now let us examine the theory of plastic flow with the von Mises condition. In accordance with the latter, the following is valid for the slip planes of the mechanical model

$$\varepsilon_n = \sigma_n/K, \quad \Delta\gamma_n^p = \Delta\lambda\tau_n. \tag{8}$$

The increment of plastic shear $\Delta\gamma_n^p$ is formed in the direction of τ_n. As before, simple extensions occur in the directions normal to the slip planes.

The slip planes in the mechanical model are octahedral planes. A change in the principle axes of the stress tensor leads to healing of the old system of slip planes and the formation of a new system in which the equations describing deformation again have the form (7)-(8). Thus, the theories being analyzed do not consider the loading history and they do not account for the old system of planes in either elastic or plastic deformation.

Let us turn our attention to the main relations of the theory of plastic flow with the Tresca yield condition. In accordance with this theory, as before, $\varepsilon_n = \sigma_n/K$, and elastic simple extension occurs along normals to the octahedral planes. However, the increment in the plastic shear strains takes place not along the shearing force τ_n, but along its projections in preassigned directions: in the state of incomplete plasticity, along the vector [see (3)] $\boldsymbol{\alpha} = (\sqrt{3/2})(n_1\mathbf{e}_1 - n_3\mathbf{e}_3)$, $|\boldsymbol{\alpha}| = 1$, in the state of complete plasticity, along $\boldsymbol{\alpha}$ and the direction formed by a ±60° angle from $\boldsymbol{\alpha}$.

There is nothing significant about these directions in the octahedral planes. Instead, we can take other directions – such as those determined by the yield condition in [34]. Here, a hexagon is replaced by a dodecagon. Thus, the Tresca condition in the model is a linear approximation of the von Mises condition, and the theories based on the former are approximations of corresponding theories constructed on the basis of the von Mises yield condition.

Different arguments are made to explain the use of the Tresca condition in elastoplastic models. In particular, a volumetric stress-strain state is represented in the form of a superposition of three plane strains [5, 7]. However, there is one inherent contradiction to this approach. We will discuss it briefly. Let there be three plane-strain states:

$$\begin{aligned} &\varepsilon_1' - \varepsilon_2' = (\sigma_1 - \sigma_2)/A, \quad \varepsilon_1' + \varepsilon_2' = (\sigma_1 + \sigma_2)/B, \quad \varepsilon_3' = 0; \\ &\varepsilon_1'' - \varepsilon_3'' = (\sigma_1 - \sigma_3)/A, \quad \varepsilon_1'' + \varepsilon_3'' = (\sigma_1 + \sigma_3)/B, \quad \varepsilon_2'' = 0; \\ &\varepsilon_2''' - \varepsilon_3''' = (\sigma_2 - \sigma_3)/A, \quad \varepsilon_2''' + \varepsilon_3''' = (\sigma_2 + \sigma_3)/B, \quad \varepsilon_1''' = 0 \end{aligned} \tag{9}$$

(1, 2, 3 are the principal axes of the stress tensor and A and B are the as-yet-unknown rigidity moduli of the material). We will assume that all of these states occur independently of one another. We designate the total strains as $\varepsilon_k = \varepsilon_k' + \varepsilon_k'' + \varepsilon_k'''$ (k = 1, 2, 3). To select the parameters A and B, we need to connect the strains ε_k with the stresses associated with Hooke's law. Having performed the necessary operations, we obtain

$$1/A = (1 + 2\nu)/(2E), \ 1/B = (1 - 2\nu)/(2E). \tag{10}$$

The volumetric stress-strain state is decomposed into three plane strains. Each of these states has prismatic structures, simple shears, and extensions. The difference between the states of incomplete and complete plasticity is fairly clear [4-8]. Nonetheless, this deformation scheme is inconsistent. If additional orthogonal loading is done from the state (9), then by Hooke's law we should have

$$\Delta\varepsilon_{12} = \frac{1+\nu}{E}\Delta\tau_{12}, \quad \Delta\varepsilon_{13} = \frac{1+\nu}{E}\Delta\tau_{13}, \quad \Delta\varepsilon_{23} = \frac{1+\nu}{E}\Delta\tau_{23}.$$

A contradiction develops from this. On the one hand, the shear modulus $A = 2E/(1 + 2\nu)$ by virtue of (9)-(10), while on the other hand the same equations lead to $A = E/(1 + \nu)$. Thus, the material is nonisotropic even though it was initially supposed to be isotropic. Since the material cannot be deformed elastically according to Eqs. (9)-(10), we find that, in this scheme, plastic deformation [5, 7, 35] will not be preceded by elastic strain.

Tests involving simple shearing of materials (see Fig. 2b), such as the torsion of thin-walled cylindrical tubes, are of fundamental importance for constructing a mechanical model of a medium. They can be used to determine the characteristic function of simple shear $\tau = \tau(\gamma)$, where the measured value of γ is shown in Fig. 2b and includes a component of the rigid-rotation vector [33]. Also, these tests, when conducted to the point where strain localization is visually observable, give information on the orientation of the slip planes in simple shear. If the slip planes coincide with the direction of the applied shearing force τ, then the above mechanical model is valid. Otherwise (see [24] for example), the investigated isotropic material needs to be examined as a rock. In this case, it is recommended that the mechanical model and constitutive equations derived in [10] be used to describe it.

We make one more important observation. As is known, the maximum shear stress is also characterized by the plane in which it acts. Nevertheless, by virtue of (3), the difference $(\sigma_1 - \sigma_3)/2$ goes into the determination of the shear stress in any other planes. Thus, it is not identical to the maximum shear stress. On the basis of (1)-(3), the quantity $\varepsilon_2' = \varepsilon_2 - (\varepsilon_1 + \varepsilon_3)/2$ [4] plays the role of the projection of the shear strain. Finally, the block structure of an anisotropic material is established from analysis of the natural elastic states [31, 32], as was suggested in [12].

LITERATURE CITED

1. S. B. Batdorf and B. V. Budyanskii, "Mathematical theory of plasticity based on the concept of slip," Mekhanika, No. 1 (1962)
2. M. Ya. Leonov, Mechanics of Deformation and Fracture: Physico-Mathematical Principles of a Theory [in Russian], Ilim, Frunze (1981).
3. A. N. Mokhel' and R. L. Salganik, "Toward a theory of plastic deformation of strain-hardening materials," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 5 (1976).
4. E. I. Shemyakin, "Anisotropy of the plastic state," ChMMSS, 4, No. 4 (1973).
5. S. A. Khristianovich, "Elastoplastic deformation of a plastic material," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 4 (1974).
6. S. A. Khristianovich and E. I. Shemyakin, "Toward a theory of ideal plasticity," Inzh. Zh. Mekh. Tverd. Tela, No. 4(1967).
7. A. M. Kovrizhnykh, "Variant of plastic flow theory based on the shear mechanism of deformation," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1982).
8. V. A. Babakov, "Model of a plastic body with internal friction and dilatation," Fiz. Tekh. Probl. Razrab. Polezn. Iskop., No. 3 (1987).
9. A. Nadai, Plastic Deformation and Fracture of Solids [Russian translation], IL, Moscow (1954).
10. A. I. Chanyshev, "Elasticity relations for rocks. Strain theory of plasticity," Fiz. Takh. Probl. Razrab. Polezn. Iskop., No. 1 (1986).
11. A. F. Revuzhenko, "Deformation of a granulated medium," Fiz. Tekh. Probl. Razrab. Polezn. Iskop., No. 3 (1980); No. 5 (1981); No. 4 (1982); No. 6 (1983).
12. A. F. Revuzhenko, A. I. Chanyshev, and E. I. Shemyakin, "Mathematical models of elastoplastic bodies," in: Important Problems of Computer Mathematics and Mathematical Modeling [in Russian], Nauka, Novosibirsk (1985).
13. A. I. Imamutdinov, "Deformation of a plastic strain-hardening material during complex loading," Din. Sploshnoi Sredy, No. 34 (1978).
14. A. I. Chanyshev, "Model of the plastic deformation of rocks during complex loading (three-dimensional case)," Fiz. Tekh. Probl. Razrab. Polezn. Iskop., No. 1 (1985).
15. N. N. Malinin, Applied Theory of Plasticity and Creep [in Russian], Mashinostroenie, Moscow (1975).
16. A. M. Zhukov, "Ductility properties and fracture of steel in a biaxial stress state," Inzh. Sb., 20 (1954).
17. A. M. Zhukov, "Complex loading and the theory of plasticity of isotropic materials," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 8 (1955).
18. E. A. Davis, "Combined tension-torsion test with fixed principle directions," J. Appl. Mech., 22, No. 3 (1955).
19. V. V. Novozhilov, "On plastic dilatation," Prikl. Mat. Mekh., 29, No. 4 (1965).
20. J. Friedel, Dislocations, Pergamon Press, New York (1964).
21. V. A. Kuz'menko, New Schemes of Deformation of Solids [in Russian], Naukova Dumka, Kiev (1973).
22. A. F. Revuzhenko and E. I. Shemyakin, "Transverse strain effect in continuum mechanics," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1974).
23. D. Rice, "Localization of plastic strain," in: Theoretical and Applied Mechanics. Transactions of the 14th International Congress. IUTAM [Russian translation], Mir, Moscow (1979).
24. A. F. Revuzhenko, S. B. Stazhevskii, and E. I. Shemyakin, "Mechanism of deformation of a granulated material with large shear strains," Fiz. Tekh. Probl. Razrab. Polezn. Iskop., No. 3 (1974).
25. A. F. Revuzhenko, S. B. Stazhevskii, and E. I. Shemyakin, "Asymmetry of plastic flow in convergent axisymmetric channels," Dokl. Akad. Nauk SSSR, 246, No. 3 (1979).
26. A. F. Revuzhenko and E. I. Shemyakin, "Plane strain of strain-hardening and strain-softening plastic materials," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1977).

27. J. W. Rudnicki and J. R. Rice, "Condition for the localization of deformation in pressure-sensitive dilatant materials," J. Mech. Phys. Solids, 23, No. 6 (1975).
28. L. V. Nikitin and E. I. Ryzhak, "Fracture of rock with internal friction and dilatation," Dokl. Akad. Nauk SSSR, 230, No. 5 (1976).
29. V. N. Nikolaevskii, "Determination of the equation of plastic deformation of a granulated medium," Prikl. Mat. Mekh., 35, No. 6 (1971).
30. I. A. Garagash, "Stability and fracture of rock masses with applications in the mechanics of earthquake preparation," Author's Abstract of Doctor of Physico-Mathematical Dissertation, IFZ Akad. Nauk SSSR, Moscow (1985).
31. Ya. Rykhlevskii, "On Hooke's law," Prikl. Mat. Mekh., 48, No. 3 (1984).
32. A. I. Chanyshev, "On the plasticity of anisotropic bodies," Zh. Prikl. Mekh. Tekh. Fiz., No. 2 (1984).
33. N. I. Muskhelishvili, Some Basic Problems of the Mathematical Theory of Elasticity [in Russian], Izd. Akad. Nauk SSSR, Moscow (1949).
34. D. D. Ivlev, Theory of Ideal Plasticity [in Russian], Nauka, Moscow (1966).
35. A. M. Kovrizhnykh, "Variant of the theory of plastic deformation of rocks," Fiz. Tekh. Probl. Razrab. Polezn. Iskop., No. 1 (1983).
36. N. I. Bezukhov, Principles of the Theory of Elasticity, Plasticity, and Creep [in Russian], Vysshaya Shkola, Moscow (1968).

CRACK GROWTH IN METALS AT ELEVATED TEMPERATURE

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Creep is the capacity of all solids to undergo irreversible deformation under constant loads due to the thermal motion and directional migration under load of the main sources of such deformation: inclusions, voids, dislocations, and microcracks. The latter, coalescing at the final stage of creep, form a macrocrack which separates the structural element. Creep in metals usually becomes noticeable at temperatures greater than one-third of the melting point (in K).

The phenomenological approach to creep is semi-empirical and is based on many additional assumptions regarding irreversible (plastic) strains that have been justified on the basis of experiments for specific materials under certain conditions [1].

The new approach being taken to fracture mechanics in creep and plasticity consists of the following: the material is considered to be linearly or nonlinearly elastic, while the sources of irreversible strain are examined in explicit form [2, 3]. In this approach, irreversible strain is calculated as being the result of the nucleation, movement, and growth of these sources, while fracture is represented by a certain calculable critical moment of instability of plastic strain. It is possible to examine different deterministic and statistical systems of sources by using the methods of the theory of diffusion and migration to study their motion and development [2, 3].

Since the 1970s and the publication of [4], the growth of creep cracks in metals has been subjected to massive experimental study within the framework of classical fracture mechanics on the basis of stress intensity factors [5] and invariant energy integrals [6-18].

As was shown in [19], the δ_K-concept in the Leonov—Panasyuk—Dugdale model follows from the general energy-based Γ_c-concept. The analog of the δ_K-concept for linear viscoelastic materials was developed in [20]. In this case, the Γ_c- and δ_K-concepts differ.

In the theory of elasticity, invariant integrals were first found by the Maxwell method by Eshelby in 1951. The basic invariant energy integral (more general than Eshelby's) used as a criterion in the theory of fracture was obtained directly from the conservation law in [6] for an arbitrary solid. Obtained with it was the solution of the problem

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